Rossmoyne Senior High School

MATHEMATICS
METHODS
UNIT 1
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised material. If you have any unauthorised material with you, hand
it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	11	11	100	85	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

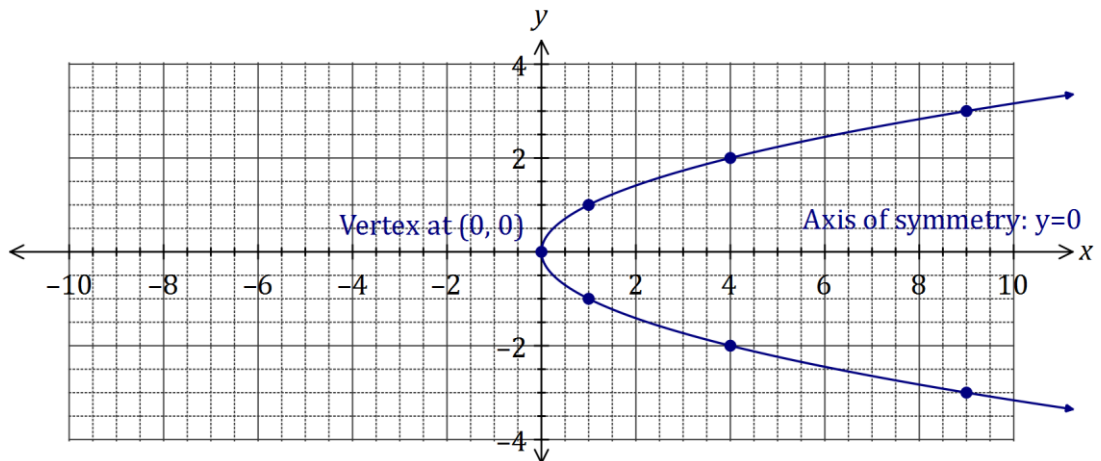**Section One: Calculator-free** **35% (52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

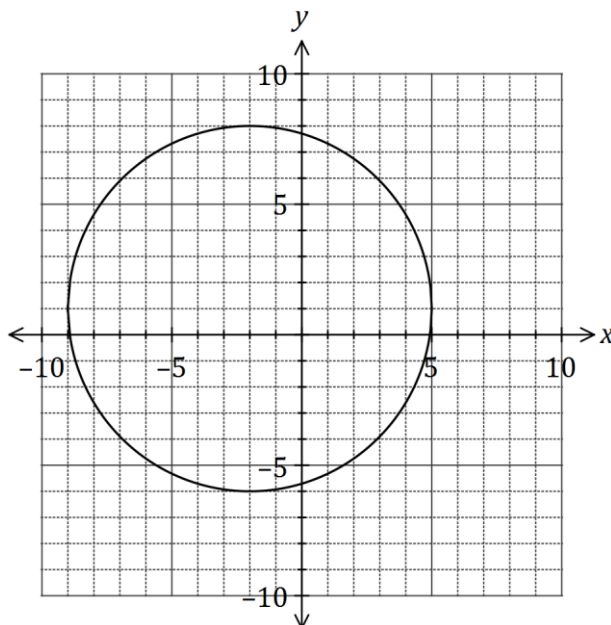Working time: 50 minutes.

Question 1 **(6 marks)**

(a) On the axes below, sketch the graph of the relation $y^2 = x$, labelling all key features with their coordinates or equations. (3 marks)

Solution
See graph
Specific behaviours
✓ smooth curve through at least 5 out of 7 marked points
✓ labels vertex
✓ labels axis of symmetry

(b) Determine the equation of the circle shown below. (3 marks)

Solution
$(x + 2)^2 + (y - 1)^2 = 49$
Specific behaviours
✓ uses standard form for circle
✓ uses correct centre
✓ uses correct radius

Question 2   **(9 marks)**

(a)   The point $M(8, 1)$ is the midpoint of A and $B(20, 7)$. Determine the coordinates of A.

(2 marks)

Solution
$$\frac{20 + x}{2} = 8 \text{ and } \frac{7 + y}{2} = 1$$ $$x = -4, y = -5 \Rightarrow A(-4, -5)$$
Specific behaviours
✓ writes equations
✓ states coordinates of A

(b)   A relationship between x and y is given by $x = 2y - 3$.

(i)   Determine y when $x = 25$.   (1 mark)

Solution
$25 = 2y - 3 \Rightarrow y = 14$
Specific behaviours
✓ states value

(ii)   State, with justification, whether x is a function of y.   (2 marks)

Solution
Yes.
All values of the domain (y) are mapped to one value of the range (x).
Specific behaviours
✓ states yes
✓ reasoning includes one output for each input

(c) A straight line passes through points $C(2, -5)$ and $D(-2, 2)$. Determine the equation of the straight line that is perpendicular to this line and passes through C, expressing your answer in the form $ax + by + c = 0$, where a, b and c are integers. (4 marks)

Solution
$$m_{CD} = -\frac{7}{4} \Rightarrow m_\perp = \frac{4}{7}$$ $$y - (-5) = \frac{4}{7}(x - 2) \Rightarrow -4x + 7y + 43 = 0$$
Specific behaviours
✓ determines gradient of CD
✓ determines perp. Gradient
✓ writes equation of line
✓ writes equation in required form

See next page

Question 3   **(7 marks)**

Solve each of the following equations for the variable x.

(a) $5x - 6 = 9$

Solution
$5x - 6 = 9$
$5x = 15$
$x = 3$
Specific behaviours
✓ moves 6 to other side
✓ solves

(b) $3(1 - x) + 4 = 2(2x - 7)$.   (2 marks)

Solution
$3 - 3x + 4 = 4x - 14$
$21 = 7x \Rightarrow x = 3$
Specific behaviours
✓ expands
✓ simplifies and solves

(c) $\dfrac{4}{x} = 3 + x$.   (3 marks)

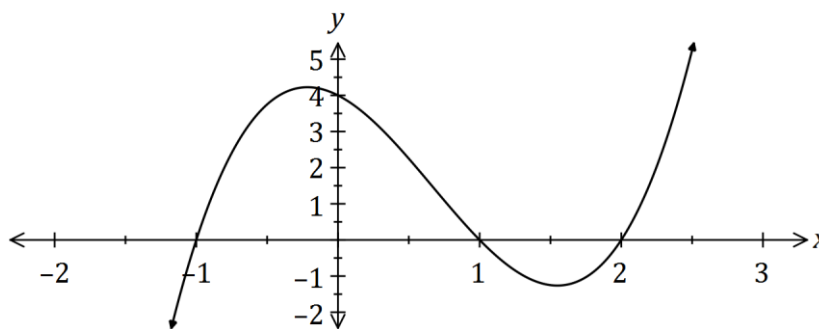| Solution |
|---|
| $4 = 3x + x^2 \Rightarrow x^2 + 3x - 4 = 0$ |
| $(x + 4)(x - 1) = 0$ |
| $x = -4, x = 1$ |
| **Specific behaviours** |
| ✓ equates to zero |
| ✓ factors |
| ✓ solves |

Question 4 (8 marks)

(a) Solve the equation $x^3 = 7x^2 + 30x$. (4 marks)

Solution
$x^3 - 7x^2 - 30x = 0$
$x(x^2 - 7x - 30) = 0$
$x(x - 10)(x + 3) = 0$
$x = 0, x = 10, x = -3$
Specific behaviours
✓ equates to zero
✓ factors out x
✓ factors quadratic
✓ states all solutions

(b) The graph of $y = ax^3 + bx^2 + cx + d$ is shown below. Determine the values of the constants a, b, c and d. (4 marks)

Solution
$y = a(x + 1)(x - 1)(x - 2)$
$x = 0, y = 4 \Rightarrow 4 = 2a, a = 2$
$y = 2(x^2 - 1)(x - 2) = 2(x^3 - 2x^2 - x + 2)$
$a = 2, b = -4, c = -2, d = 4$
Specific behaviours
✓ writes factors from roots
✓ determines a
✓ expands
✓ states all values

Question 5   **(6 marks)**
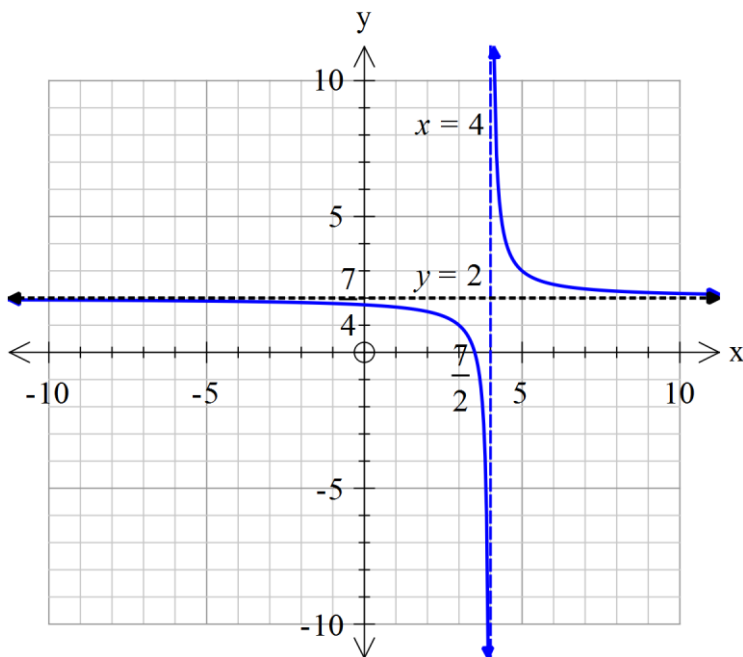
Let $f(x) = \dfrac{1}{x}$

(a) Determine the new function if $f(x)$ is translated 4 units to the right and 2 units up.

(2 marks)

Solution
$y = \dfrac{1}{x-4} + 2$
Specific behaviours
✓ use x -4   ✓ uses + 2

(b)Sketch the transformed function in (a) above on the axes below showing all features.

(4 marks)

Solution
Specific behaviours
✓ draws in horizontal asymptote and labels equation   ✓ draws in verticall asymptote and labels equation   ✓ states y intercept   ✓ states x intercept

See next page

Question 6   **(8 marks)**

(a)   For the graph with equation $y = (x+1)(x-3)$, determine the coordinates of

   (i)   all axes intercepts.   (2 marks)

Solution
y-intercept: $(0, -3)$. x-intercepts: $(-1, 0)$ and $(3, 0)$
Specific behaviours
✓ y-intercept
✓ x-intercepts and all answers written as coordinates

   (ii)   the turning point.   (2 marks)

Solution
Line of symmetry: $x = 1$
$y = (1+1)(1-3) = -4$
TP at $(1, -4)$
Specific behaviours
✓ x-coordinate
✓ turning point in coordinate form

(b)   The graphs of three quadratic functions with discriminants of 0, 1 and -2 are shown below.
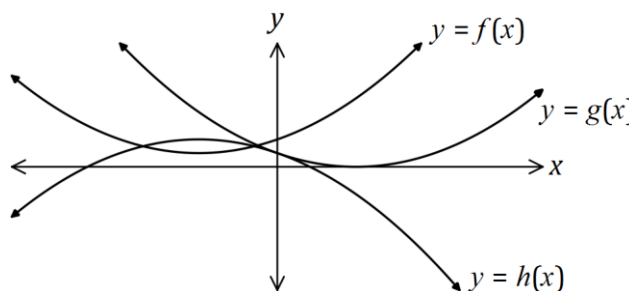
Solution (i)
see table
Specific behaviours
✓ at least one correct
✓ all correct

   (i)   Underneath each function in this table, write the value of it's discriminant.
                                                                                        (2 marks)

Function	$f(x)$	$g(x)$	$h(x)$
Discriminant	-2	0	1

   (ii)   Clearly explain your choices in part (i).   (2 marks)

Solution (ii)
f has no roots, g has 1 root and h has two roots. Discriminant is: -ve, no roots; 0, one root; and +ve, two roots.
Specific behaviours
✓ states number of roots of each function
✓ explains use of discriminant

Question 7 **(8 marks)**

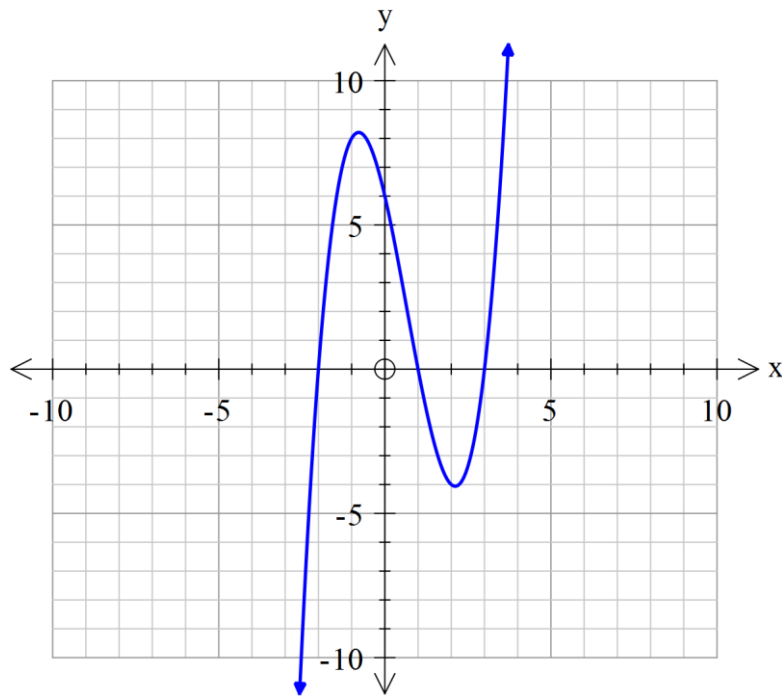(a) Divide the polynomial $x^3 - 2x^2 - 5x + 6$ by $x - 1$ (4 marks)

$$
\begin{array}{r}
x^2 - x - 6 \\
x - 1{\overline{\smash{\big)}\,x^3 - 2x^2 - 5x + 6}} \\
x^3 - x^2 \\
0 - x^2 - 5x \\
0 - x^2 + x \\
0 + 0 - 6x + 6 \\
0 + 0 - 6x + 6 \\
0
\end{array}
$$

Solution (ii)
.
Specific behaviours
✓ sets up table for long division
✓ determines first term in quotient
✓ determines complete quotient
✓has zero remainder

(b) Hence sketch $y = x^3 - 2x^2 - 5x + 6$ showing x and y intercepts. (4 marks)

Solution (ii)
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Specific behaviours
✓ fully factorises to obtain three x intercepts
✓ states y intercept
✓ correct shape
✓ local max and min placed in correct interval, though not drawn to scale

Additional working space

Question number: _____